$$W_{(x)} = -\frac{\pi\epsilon x^2 J_1\{[K^2a^2(\epsilon-1)+x^2]^{1/2}\}[J_0{}^2(x)+N_0{}^2(x)]}{2[K^2a^2(\epsilon-1)+x^2]^{1/2}J_0\{[K^2a^2(\epsilon-1)+x^2]^{1/2}\}} - \frac{\pi x}{2}[J_1(x)J_0(x)-N_1(x)N_0(x)], \tag{34}$$

$$M(\theta) = J_1[Ka(\epsilon-\cos^2\theta)^{1/2}]\epsilon\sin\theta[J_0{}^2(\beta)+N_0{}^2(\beta)] - (\epsilon-\cos^2\theta)^{1/2}J_0[Ka(\epsilon-\cos^2\theta)^{1/2}] \cdot [J_1(\beta)J_0(\beta)+N_1(\beta)N_0(\beta)], \tag{35}$$

$$\beta = Ka\sin\theta. \tag{36}$$

$$\text{Total power radiated} = \int_0^{\pi} L(\theta)d\theta. \tag{37}$$

Eqs. (30) and (31) are computed numerically on the computer for $\epsilon = 2.49$ for various values of Ka. These numerical results are plotted in Fig. 2. From these results it is clear that

1) The excitation efficiency for $Ka > 3$ is quite high (above 70 per cent). Hence it is an efficient way of excitation. The excitation efficiency does not depend critically upon the precise values of Ka.
2) The larger the normalized cross section of the dielectric rod (Ka) is, the higher is the excitation efficiency.
3) The excitation efficiency curve resembles very closely the efficiency curves obtained in exciting a corrugated surface[4] or a dielectric slab.[5]
4) The excitation efficiency can be improved by flaring out the edge of the metallic waveguide at $z=0$ to accommodate a larger dielectric rod without causing a second mode to propagate inside the metallic waveguide.

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[4] A. L. Cullen, "The excitation of plane surface waves," *Proc. IEE*, Monograph No. 93, vol. 101, pt. 4; 1954.

[5] C. M. Angulo and W. S. C. Chang, "On the Excitation of Surface Waves, Part I, II, and III," Div. of Eng., Brown University, Providence, R. I., Scientific Reps. No. AF 1391/3 to AF 1391/5, pp. 67–72; 1956.

An Investigation of the Properties of Germanium Mixer Crystals at Low Temperatures*

L. K. ANDERSON† AND A. HENDRY‡

Summary—**Experimental determinations of the noise temperature ratio, IF resistance, and conversion loss of 1N263 germanium mixer diodes operated in an X-band receiver are presented as a function of mixer temperature for the range −196°C to 27°C. No improvement in receiver noise factor was obtained by cooling the mixer to −196°C; however an improvement of 0.3 to 0.6 db was observed by cooling to a temperature in the region −100°C to −50°C. The exact value of the improvement and the optimum temperature depends on the individual crystal, as well as on dc bias and local oscillator drive.**

I. INTRODUCTION

IT has been suggested,[1] largely on theoretical grounds, that the over-all noise factor of a superheterodyne receiver, employing a germanium crystal mixer, may be improved by cooling the crystal to a temperature substantially below the ambient temperature. The work discussed in this paper was carried out in an effort to verify this prediction, and also to determine how the various crystal parameters, such as IF resistance and noise temperature ratio, vary with temperature in the range from room temperature to the boiling point of nitrogen (about −196°C). The work was carried out at 9375 mc, using type 1N263 germanium diodes.

II. MEASUREMENT OF CRYSTAL AND SYSTEM PARAMETERS

Fig. 1 is a block diagram of the apparatus, with which the following parameters may be determined: over-all receiver noise factor, IF amplifier noise factor, and the noise temperature ratio, IF resistance, and conversion loss of the crystal mixer. The over-all receiver noise factor and the IF amplifier noise factor are determined by standard methods, *e.g.*, fluorescent lamp waveguide noise source followed by a precision waveguide attenuator for the over-all noise factor, and a temperature limited noise diode with 3-db attenuator in the IF amplifier for the IF noise factor.

The methods used for the measurement of the crystal parameters are largely those described by Torrey and Whitmer.[2]

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[1] G. C. Messenger, "Cooling of microwave crystal mixers and antennas," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-5, pp. 62–63; January, 1957.

[2] H. C. Torrey and C. A. Whitmer, "Crystal Rectifiers," M.I.T. Rad. Lab. Ser., McGraw-Hill Book, Co., Inc., New York, N. Y., vol. 15, pp. 223–226; 1948.

A. *Noise Temperature Ratio*

For the measurement of the noise temperature ratio of the crystal, the output meter of the receiver was calibrated by means of a resistance cartridge in place of the crystal. For calibration, the resistance cartridge was made noisy by means of a current I passing through the noise diode. Such a resistor has a noise temperature ratio given by:[3]

$$t = \frac{T}{T_0} + 20IR \qquad (1)$$

where

t = noise temperature ratio,
T = absolute temperature of resistance cartridge,
$T_0 = 290°\text{K}$, the standard temperature,
R = resistance of cartridge, which serves as the noise diode plate load.

The use of a Roberts coupling circuit[4] allowed calibration of the output meter directly in terms of crystal noise temperature ratio without knowledge of the IF resistance. Provided that the resistance was in the range 100–200 ohms, and that the noise temperature ratio did not exceed 2, t could be determined within about 5 per cent by this method.

B. *Crystal IF Resistance*

The IF resistance was determined from the change in noise power output of the amplifier occasioned by a specified change in diode plate current, using the crystal as a diode load. A calibration was made using resistor cartridges of known resistance.

C. *Conversion Loss*

The conversion loss may be calculated from the expression for over-all noise factor

$$L = \frac{F_{\text{rec}}}{F_{\text{IF}} + (t - 1)} \cdot \qquad (2)$$

However, the need for separate knowledge of F_{IF} and t may be circumvented by a measurement using the crystal as a plate load for the noise diode in place of the usual load resistor in the standard measurement of F_{IF}. With this change, the quantity measured is essentially the denominator of (2), and it may then be shown[5] that the conversion loss is given by:

$$L = \frac{F_{\text{rec}}}{20I_D R_{\text{IF}}} \qquad (3)$$

where I_D is the diode plate current to double the noise power output of the IF amplifier.

Note: The accuracy of the value of R_{IF} determined in Part B above is limited by uncertainties in the measurement of the change of output power. A more accurate value may be obtained by combining (2) and (3) to obtain:

$$R_{\text{IF}} = \frac{F_{\text{IF}} + t - 1}{20I_D} \cdot \qquad (4)$$

Since F_{IF} is a slowly varying function of R_{IF}, the less accurate value of R_{IF} determined in Part B may be used to obtain F_{IF} for insertion in (4).

III. APPARATUS AND EXPERIMENTAL TECHNIQUES

A block diagram of the entire setup is shown in Fig. 1. Fig. 2 is a photograph of the crystal mount and cooling bath.

A. *RF Section*

All of the RF components, with the exception of the crystal mount, are completely conventional. The TR tube in the local oscillator line was used as a noise filter, and was tuned to the local oscillator frequency. At the signal frequency the attenuation of the filter was sufficient to reduce the LO noise to insignificance.

The crystal mount used was a simple broad-band type, primarily designed for detector use. The VSWR of the mount itself, which was never more than 2 for any of the crystals tested, was reduced to better than 1.1 for every determination by means of a slide-screw tuner ahead of the crystal mount. The use of this type of holder was established by two requirements: 1) The mount had to be easily cooled and so had to be small, and 2) it had to be capable of being sealed against the entry of the coolant (liquid nitrogen) and circulating air; thus no internal tuning adjustments could be used.

Fig. 3 is a sketch of the crystal mount assembly. The crystal mount was cooled by immersion in liquid nitrogen (boiling point: $-195.8°\text{C}$) to the level indicated. In order to prevent condensation of oxygen in the crystal mount it was necessary to seal it off from the rest of the waveguide by means of a thin celluloid window.

A one-foot section of "Glas-guide" was placed between the crystal mount and the rest of the waveguide system. This waveguide, with a thermal conductivity only one-hundredth that of equivalent brass waveguide, provided the necessary thermal isolation. In addition, the long vertical section of "Glas-guide" provided a column of stagnant air which, except on very humid days, prevented the appearance of any significant amounts of frost inside the waveguide despite a considerable accumulation on the outside.

With the crystal mount at the desired temperature, the dc bias was set to the desired value with no RF excitation, and then the local oscillator drive was adjusted to give the desired total crystal current. The crystal mount VSWR, measured at the local oscillator frequency (9375 mc), was then reduced to less than 1.05 by means of the slide-screw tuner.

Details of the crystal bias supply are given in Fig. 4.

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[3] *Ibid.*, p. 230.

[4] *Ibid.*, p. 231.

[5] T. Nicoll, "Noise in silicon microwave diodes," *Proc. IEE*, pt. 3, vol. 101, pp. 317–324; September, 1954.

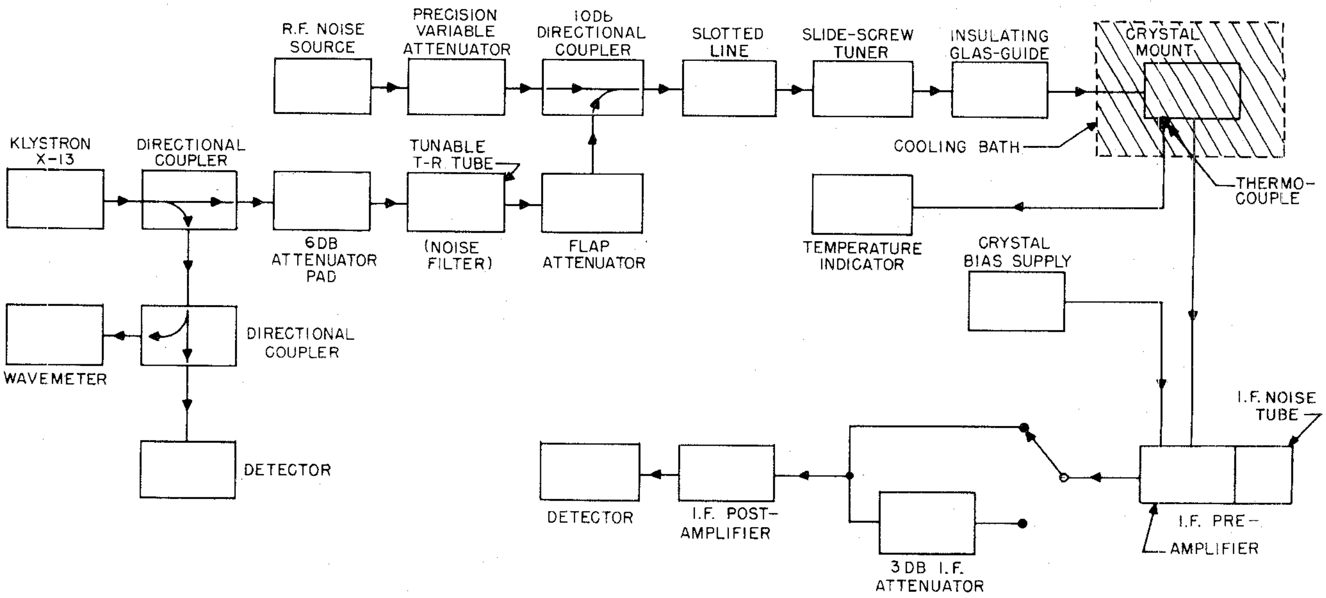

Fig. 1—Block diagram of test set-up.

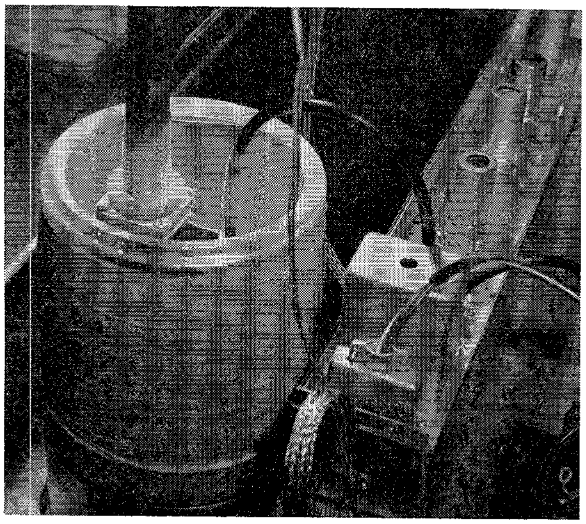

Fig. 2—Crystal mount and cooling bath.

B. IF Section

The IF amplifiers constructed were of conventional design except for the Roberts coupling circuit used at the input to the preamplifier.

The IF amplifier bandwidth is 1 mc, while the center frequency is 30 mc.

The detector circuit employed is approximately square law, but for accurate determination of relative output power levels, as required in the determination of R_{IF}, the IF power was measured directly at 30 mc by a barretter bridge at the output of the post-amplifier.

IV. Experimental Results

A. Comparison of Over-all Receiver Noise Factor at Room Temperature and at $-196°C$

Using the gas discharge RF noise source, the over-all receiver noise factor F_{rec} was measured: 1) at room temperature, 2) at −196°C, and 3) at room temperature again. This was done for twelve 1N263 crystals. The results of the measurements are given in Table I.

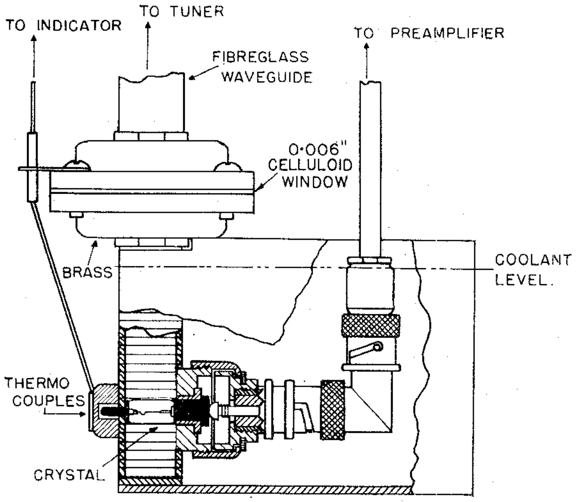

Fig. 3—Crystal holder assembly.

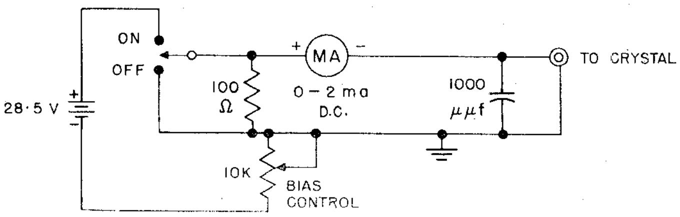

Fig. 4—Crystal bias supply.

For the above determinations, the crystals were biased as follows:

At 27°C: dc bias current 0.9 ma, with local oscillator drive adjusted to give 1.8-ma total crystal current.

At −196°C: dc bias current 0.3 ma, with local oscillator drive adjusted to give 1.8-ma total crystal current.

TABLE I

COMPARISON OF NOISE FACTORS AT 27°C AND −196°C

Crystal No.	F_{rec} db		
	27°C	−196°C	27°C
1	10.9	11.1	10.9
2	11.7	12.9	11.7
3	11.4	11.6	11.5
4	10.4	11.1	10.5
5	10.3	9.9	10.4
6	11.1	11.0	11.1
7	11.4	15.4	11.9
8	11.3	11.6	10.4
9	11.2	11.4	10.4
10	10.5	10.6	11.0
11	11.5	12.3	10.4
12	11.4	10.8	10.7

TABLE II

CRYSTAL PARAMETERS AT 27°C AND −196°C

Crystal No.	Temp. °C	t (ratio)	R_{IF} (ohms)	L (ratio)	$F_x = Lt$ (ratio)
1	27	1.35	118	4.41	5.95
	−196	1.99	169	3.70	7.36
	27	1.35	117	4.72	6.38
2	27	1.33	124	4.02	5.35
	−196	1.86	177	2.97	5.52
	27	1.33	126	3.92	5.22

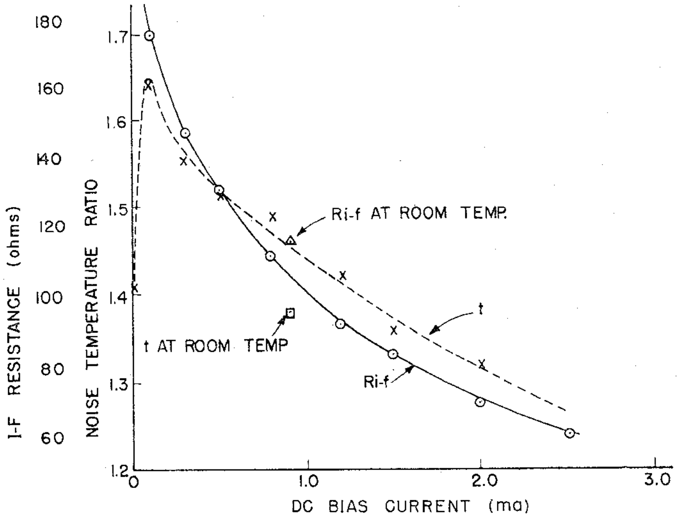

Fig. 5—Crystal IF resistance and noise temperature ratio as a function of bias at −196°C.

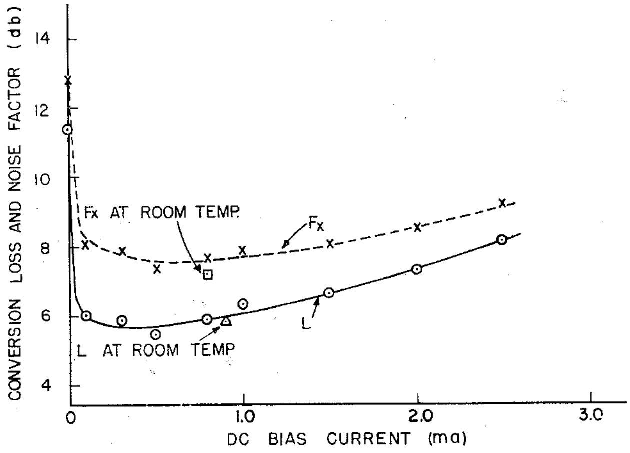

Fig. 6–Crystal conversion loss and noise factor as a function of bias at −196°C.

The room temperature bias is that recommended by the crystal manufacturer, while the dc bias used at −196°C is that value which was found by experiment to produce the minimum average value of F_{rec} for the twelve crystals in the group, when the local oscillator excitation was fixed at a value sufficient to provide 1.8-ma total current.

With the exception of crystal no. 7, these results indicate no consistent trend of either improvement or deterioration, although it is of interest to note that the worst crystal at room temperature suffered a deterioration, while the best showed an improvement upon cooling. Most of the others showed little change.

B. Comparison of Crystal Parameters at Room Temperature and −196°C

Using the method outlined in Part A above, extensive measurements were made on three 1N263 crystals. One of these showed an excessive change in room temperature parameters after the cooling cycle, and therefore the results for this crystal were discarded. Data obtained on the remaining two are presented in Table II.

The bias conditions were as follows:

At 27°C: dc bias current 0.9 ma, with local oscillator drive adjusted to give 1.8-ma total crystal current;

At −196°C: dc bias current 0.3 ma, with local oscillator drive adjusted to give 2.0-ma total crystal current.

(Considering only the two crystals for which data are given in Table II, a total bias current of 2.0 ma rather than 1.8 ma yielded the minimum average value of F_{rec}.)

In both cases, although there was a slight reduction in the conversion loss at −196°C, the increase in the noise temperature ratio resulted in a net deterioration of the crystal noise factor.

C. Variation of Crystal Parameters at −196°C with DC Forward Bias

It was felt that a combination of bias and local oscillator drive differing from the values used in obtaining the preceding data might result in some net improvement of crystal noise factor. The effect of the variation of dc forward bias was investigated for a single crystal. The local oscillator level used was about 0.5 milliwatt, a value which had been found to be sufficiently close to optimum on the basis of some rough preliminary measurements. The results of these measurements are shown in Figs. 5 and 6. Room temperature values for these parameters, with 0.9-ma current bias, are also indicated.

These curves indicate that the optimum value of dc

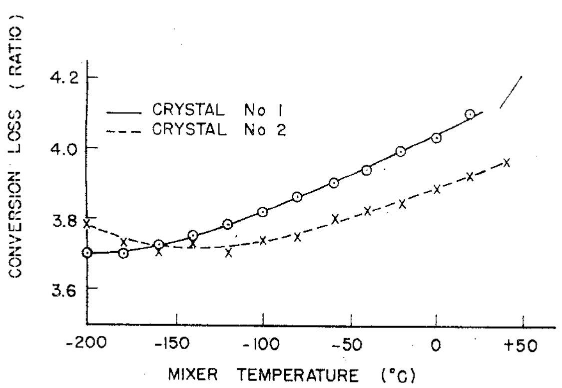

Fig. 7—Crystal conversion loss as a function of temperature.

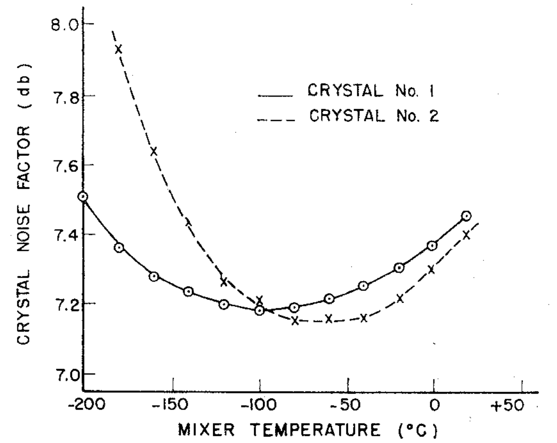

Fig. 8—Crystal noise factor as a function of temperature.

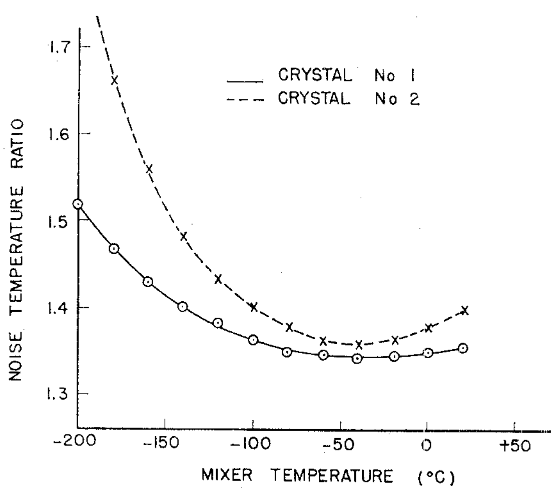

Fig. 9—Crystal noise temperature ratio as a function of temperature.

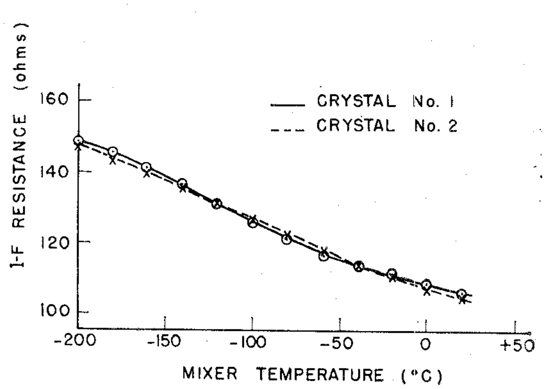

Fig. 10—Crystal IF resistance as a function of temperature.

bias current is between 0.5 and 0.6 ma, with the exact value being relatively uncritical. However, the best noise factor that can be obtained at −196°C is still inferior to that obtained at room temperature with the recommended values of bias and local oscillator drive.

D. Variation of Crystal Parameters with Temperature

Although on the basis of simple theoretical considerations[1] the crystal noise factor should decrease steadily with decreasing temperature, it was felt that in practice some optimum intermediate temperature might exist. Accordingly, the various crystal parameters as a function of temperature were measured, for two different crystals, over the range from room temperature down to −196°C.

Although the dc bias required for optimum receiver noise figure is a function of temperature, this function was not known, and so it was decided to hold the dc bias constant at some compromise value. The value selected was such as to cause 0.4 ma of crystal current to flow at −1.96°C. The local oscillator level remained at 0.5 milliwatt.

The results are shown in Figs. 7–10. In Fig. 8 the crystal noise factor, $F_x = Lt$, is shown as a function of temperature. This quantity represents the noise factor of a receiver having a noise free IF amplifier ($F_{IF} = 1$ time). Both curves are seen to have minima which occur between −100 and −50°C, but the best value of F_x is, in both cases, only 0.25 db below the corresponding room temperature value.

In the determination of crystal parameters as a function of temperature, the conversion loss L and IF resistance R_{IF} varied somewhat erratically with temperature. Figs. 7 and 8 thus represent smoothed-out versions of the original curves. The cause of this erratic behavior is not known, but may be associated with discontinuous changes in the mechanical configuration of the crystal package, brought about by the steady drift in the temperature of the crystal mount as it was allowed to warm up.

In order to determine whether or not further improvements in the crystal noise factor could be made at intermediate temperatures, the parameters of two 1N263 crystals were determined as a function of local oscillator drive and dc bias at a mixer temperature of approximately −75°C. At that temperature, the crystal noise factors, $F_x = Lt$, exhibit very broad minima with best performance in the region of 0.8-ma dc bias current, and 1.8-ma total crystal current. Crystal noise factors 0.3 to 0.6 db lower than room temperature values were observed; however, the crystal parameters varied erratically at this temperature.

Some measurements were made at a mixer temperature of approximately −125°C, where the optimum operating conditions were determined to be: dc bias current 0.4 ma, total current 1.8 ma, at which improvements of 0.4 db in F_x compared to room temperature values were observed.

E. Dependence of Crystal Parameters on RF Circuitry

Using a mixer crystal mount of different design, some of the measurements made previously were repeated. These measurements, which were made first at room temperature, and later at −196°C, showed that the over-all receiver noise factor was approximately 0.7 db lower at all temperatures than the values obtained with first crystal mount, owing, apparently, to superior performance of the new crystal mount. There was, however, no change in the previously determined variations of crystal parameters with temperature. Thus it may be concluded that the failure to observe significant improvements in crystal sensitivity was not due to the performance of the particular crystal mount used.

V. Effect of Temperature Cycling on the Crystal Package

Seventeen 1N263 crystals were used in the tests, not all of them continuously. Of this number, two were ruined by the temperature cycling; *i.e.*, the noise factors became unusably high. The remainder exhibited little change in electrical properties, although small cracks appeared in the glass at the glass-to-metal seal on two of the crystals. This may cause eventual failure of these units.

VI. Conclusions

The following conclusions have been reached concerning the operation of 1N263 crystals in an X-band mixer:

1) No improvement can be made in crystal noise factor by operating the mixer at liquid nitrogen temperatures.

2) A small improvement in crystal noise factor may be possible by operating the mixer at some temperature intermediate between −196°C and room temperature. The improvement is of the order of 0.3 to 0.6 db, with the minimum value of the crystal noise factor occurring in the region between −100°C and −50°C, the exact value depending on the individual crystal, as well as on bias and local oscillator drive.

The failure to obtain significant improvement by mixer cooling is due to an increase in the noise temperature ratio of the crystal as the temperature is lowered. This suggests that flicker noise may be appreciable at 30 mc for germanium as well as for silicon mixer crystals[5] and furthermore that this excess noise is temperature dependent.

The Multiple Branch Waveguide Coupler*

JOHN REED†

Summary—**A multiple branch directional coupler is discussed for rectangular waveguide applications for series junctions. A design method is developed which is valid for any coupling ratio and any number of branch lines with perfect match and directivity. The frequency response of this type coupler is calculated with the aid of a digital computer.**

Introduction

The multiple branch waveguide directional coupler has been found to be a useful type of coupler, especially for rather tight ratios such as from zero to fifteen decibels. This paper describes the design of such a coupler and the calculation of its frequency response. The coupler is particularly desirable since the design constants are readily found and its frequency response can be calculated.

Fig. 1 shows a typical coupler with five branches. The three center connecting branch lines are made of waveguide of the same width as the main line but of reduced height c. The height of the two end branches is a different value a but the width is the same as the main line. The a and c values are normalized to the height of the main line as unity. The spacing between the center lines of adjacent branch lines is assumed to be identical and the length of all these branch lines is assumed to be the same. At first, each of these two distances will be considered to be a quarter wavelength, but for considering the frequency dependence of the coupler this restriction will be dropped. The junction effects will be disregarded, that is to say, the junctions will be regarded as pure series junctions.[1] This is strictly valid only for the case that a and c are very small, but for a first approximation it is quite valuable.

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* Manuscript received by the PGMTT, March 3, 1958; revised manuscript received, July 7, 1958.

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[1] C. G. Montgomery, R. H. Dicke, and E. M. Purcell, "Principles of Microwave Circuits," McGraw-Hill Book Co., Inc., New York, N. Y., p. 288; 1948.